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ECEN 5713 Linear System
Spring 1998
Final Exam

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Problem 1:

Determine an observable canonical form realization (in minimal order) for discrete-time system

$$ky(k+3)+\cos k y(k+2)+k^2 y(k)=e^{-k}u(k+3)+(k+1)u(k+1)+e^{-k^2}u(k)\ .$$

Notice that gain block maybe k dependent. Show the simulation diagram and its corresponding state space representation.

Problem 2:

Given

$$A(t)=\begin{bmatrix} a_{11}(t) & a_{12}(t) \\ a_{21}(t) & a_{22}(t) \end{bmatrix},$$

show that

$$\det\Phi(t,t_0)=\exp\left[\int_{t_0}^{t}\left(a_{11}(\tau)+a_{22}(\tau)\right)d\tau\right],$$

where ${\partial\Phi(t,t_0)}/{\partial t}=A(t)\Phi(t,t_0)$ and $\Phi(t_0,t_0)=I$.

(hint: begin with $\frac{\partial}{\partial t}\det\Phi(t,t_0)$)

<u>Problem 3</u>:

For a given matrix

$$A = \begin{bmatrix} \lambda & 1 & 0 & \cdots & 0 & 0 \\ 0 & \lambda & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & \lambda & 0 \\ 0 & 0 & 0 & \cdots & 0 & \lambda \end{bmatrix},$$

find $\ln A$ (i.e., $\lambda > 0$).

Problem 4:

Consider the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\alpha_n & -\alpha_{n-1} & -\alpha_{n-2} & \cdots & -\alpha_1 \end{bmatrix}.$$

Show that the characteristic polynomial of A is

$$\Delta(\lambda) = \det(\lambda I - A) = \lambda^n + \alpha_1 \lambda^{n-1} + \alpha_2 \lambda^{n-2} + \cdots + \alpha_{n-1}\lambda + \alpha_n .$$

If λ_1 is an eigenvalue of A (i.e., $\Delta(\lambda_1) = 0$), show that $\begin{bmatrix} 1 & \lambda_1 & \lambda_1^2 & \cdots & \lambda_1^{n-1} \end{bmatrix}^T$ is an eigenvector associated with λ_1.

Problem 5:

Let

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find e^{At} (hint: using Inverse Laplace transform).